Assignment No# 2

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Subject: Discrete Structures

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Assignment Title: Exploring Relations in Discrete StructuresObjective:

This assignment aims to deepen your understanding of relations in discrete mathematics by exploring various types of relations, their properties, and practical applications.

1: Binary Relations

a. Define a binary relation.

A binary relation (R) between two sets (A) and (B) is a subset of the Cartesian product (A \times B). It consists of ordered pairs ((a, b)) where (a \in A), (b \in B), and (a) is related to (b) under (R)

.**Example:** For (A = {1, 2}) and (B = {3, 4}), a binary relation could be (R = {(1, 3), (2, 4)})

.**b. Provide an example of a binary relation between two sets (A = {1, 2, 3}) and (B = {4, 5}).**

Consider the relation (R) where ((a, b) \in R) if (b = a + 3):If (a = 1), then (b = 1 + 3 = 4), so ((1, 4) \in R).If (a = 2), then (b = 2 + 3 = 5), so ((2, 5) \in R).If (a = 3), then (b = 3 + 3 = 6), but 6 is not in (B), so no pair exists for (a = 3).Thus, (R = {(1, 4), (2, 5)}).

c. Represent your relation using the Cartesian product.

The Cartesian product is:

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A \times B = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}

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The relation (R) is a subset of (A \times B):

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R = {(1, 4), (2, 5)} \subseteq A \times B].

2.Properties of Relations:

The relation $R = \{(x, y) \mid x \leq y\}$ is on the set $A = \{1, 2, 3, 4\}$.

1. Is R reflexive? Yes, because $x \leq x$ for all $x \in A$.
2. Is R symmetric? No, because $(1, 2) \in R$ but $(2, 1) \not\in R$.
3. Is R antisymmetric? Yes, because if $x \leq y$ and $y \leq x$, then $x = y$.
4. Is R transitive? Yes,because if $x \leq y$ and $y \leq z$, then $x \leq z$.

3.Equivalence Relations:

Definition:

"An equivalence relation is reflexive, symmetric, and transitive."

Examples:

- Equality ($x = y$): Reflexive ($x = x$), symmetric (if $x = y$, then $y = x$), transitive (if $x = y$ and $y = z$, then $x = z$).
- Congruence modulo n: Numbers where difference is divisible by n, satisfies all properties.
- Parallel lines: Lines parallel to each other, satisfies all properties.

4.Partial Orders:

Definition:

"A partial order is reflexive, antisymmetric, and transitive, used for ordering where not all elements are comparable."

Importance: Partial orders are crucial in discrete mathematics for modeling relationships where ordering is partial, such as in set theory (subset relations), computer science (task dependencies, data structures like lattices), and logic. They help in organizing and analyzing systems where not all elements need to be directly compared, such as in project management or database hierarchies

Examples: Set $A = \{1, 2, 3, 4\}$ with $R = \{(x, y) \mid x \leq y\}$, shown as a Hasse diagram:

4
|
3
|
2
|
1

5.Real-Life Applications:

Type: Equivalence relation, used for grouping, like students by grades.

Example: Students with the same grade (e.g., A, B) are in the same group

6.Graph Represention:

Representing the Relation as a Directed Graph

For the relation (R = {(1, 2), (2, 3), (1, 3)}) on the set (A = {1, 2, 3}):

- Each element (1, 2, 3) is a vertex in the graph.
- Each pair in (R) is shown as a directed arrow: 1 → 2, 2 → 3, and 1 → 3.
- The graph can be visualized as:

  1 → 2 → 3

  ↘

  3
- This means 1 is related to 2 and 3, and 2 is related to 3.

  Importance of Graphs for Visualizing Relations
  - Graphs make it easy to see relationships, like who is connected to whom, using arrows for direction.
  - They help spot patterns, such as chains (1 → 2 → 3) or loops, which are harder to see in a list.
  - Useful in fields like social networks (e.g., showing who follows whom) or computer science (e.g., task dependencies).